AV 3 - SÉRIES, RESÍDUOS & PÓLOS — VARIÁVEL COMPLEXA - 2017.2

ALUNO(A): GABARITO 6 ID - UFPB: __________

QUESTÕES MÚLTIPLA ESCOLHA COM RESPOSTA ÚNICA (valor 10 pontos) Nota:

01 *Assinale o valor da soma infinita* $\sum_{k=0}^{\infty} \frac{10 \cdot (z-i)^k}{(2i)^{k+1}}$, *no ponto* $z = 2i$.

(a) $-1-3i$ (b) $1-3i$ (c) $-5i$ (d) $-i$ (e) $-6i$ (f) NDR

02 *Escolha no menu o valor da integral* $\int_{|z|=1} 3z^2 \exp(-1/z)\, dz$.

(a) π (b) $-\pi i$ (c)) $-\pi$ (d) πi (e) -8π (f) NDR

03 *Certa função* $f(z)$, *analítica no disco* $|z| < 3$, *é tal que* $f(i\sqrt{3}) = 1-i$ *e* $f(-i\sqrt{3}) = 2-i$. *Assinale o valor da integral:*

$$\int_{|z|=2} \frac{f(z)\, dz}{z^2+3}$$

(a) $\frac{\pi}{\sqrt{3}}(1-3i)$ (b) $\frac{\pi}{3}(1-4i)$ (c) $-\pi/\sqrt{3}$ (d) $-\pi(1-i)$ (e) $\frac{\pi}{\sqrt{3}}(-1+2i)$ (f) NDR

04 *Se* $z_0 = i$ *é um pólo simples (de ordem 1) de certa função* $f(z)$, *assinale o valor de* $\lim_{z\to z_0} |(1-z)/f(z)|$.

(a) 0 (b) $2\pi i$ (c) πi (d) $-\pi i$ (e) ∞ (f) NDR

05 *Se* $f(z) = z^{-1}\,\mathrm{Log}(1+z)$, *então o valor de* $\mathrm{Res}(f;0)$ *é igual a:*

(a) $4i$ (b) $-2i$ (c) 0 (d) i (e) $-i$ (f) NDR

06 *Se* $f(z)$ *é analítica no anel* $0 < |z-z_0| < 3$ *e sua parte principal é* $\frac{-1}{(z-z_0)^2} + \frac{1}{i\pi\sqrt{2}(z-z_0)}$ *assinale o valor de*

$$\int_{|z-z_0|=1} f(z)\, dz + \int_{|z-z_0|=2} f(z)(z-z_0)\, dz.$$

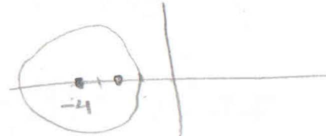
(a) $-1+2\pi i$ (b) $-\frac{1}{2}+4\pi i$ (c) $-\sqrt{2}+4\pi i$ (d) $\sqrt{2}-2\pi i$ (e) $1+2\pi$ (f) NDR

07 *Assinale o valor de* $\mathrm{Res}(f;i)$, *sabendo que* $f(z) = \frac{1}{z(z-i)^8}$.

(a) $i/32$ (b) i (c) -1 (d) $32i$ (e) 1 (f) NDR

08 *Assinale o valor da integral*

$$\int_\gamma \frac{dz}{z^2(z+2)(z+4)},$$



sobre o contorno $\gamma : |z+4| = 3$

(a) $-3\pi i/16$ (b) $3\pi i/16$ (c) $16\pi i$ (d) $-\pi i/16$ (e) 0 (f) NDR

09 *Se* γ *é o contorno* $|z| = 1$, *assinale o valor da integral*

$$\int_\gamma \frac{3z-2}{z^3+2z} dz.$$

(a) $-\pi i$ (b) $-2\pi i$ (c) $i\pi/4$ (d) πi (e) $i\pi/2$ (f) NDR

10 *Escolha no menu a função com um pólo de ordem 2 na origem.*

(a) $z^{-1}\,\mathrm{sen}\, z$ (b) $z^{-4}\,\mathrm{Log}(1+z)$ (c) $z^{-1}\cos(1/z)$ (d) $z^{-3}\,\mathrm{sen}\, z$ (e) $z^{-2}\,\mathrm{Log}(1+z)$ (f) NDR

GABARITO (PREENCHIMENTO OBRIGATÓRIO)

01	02	03	04	05	06	07	08	09	10
(a)	(a)	(a)	~~(a)~~	(a)	(a)	(a)	(a)	(a)	(a)
(b)	~~(b)~~	(b)	(b)	(b)	(b)	(b)	~~(b)~~	~~(b)~~	(b)
(c)	(c)	~~(c)~~	(c)	~~(c)~~	(c)	~~(c)~~	(c)	(c)	(c)
(d)	(d)	(d)	(d)	(d)	~~(d)~~	(d)	(d)	(d)	~~(d)~~
(e)	(e)	(e)	(e)	(e)	(e)	(e)	(e)	(e)	(e)
~~(f)~~	(f)	(f)	(f)	(f)	(f)	(f)	(f)	(f)	(f)

01 $\sum_{k=0}^{\infty} \frac{10}{2i}\left(\frac{1}{2}\right)^k = (-5i).2 = \boxed{-10i}$ (f)

02 $f(z) = 3z^2 \exp(-1/z) = 3\sum_{k=0}^{\infty} \frac{(-1)^k}{k!\,z^{k-2}} \Rightarrow \mathrm{Res}(f;0) = 3\left(-\frac{1}{6}\right) = \boxed{-\frac{1}{2}}$

Logo: $\int_{|z|=1} f(z)\,dz = 2\pi i\left(-\frac{1}{2}\right) = \boxed{-\pi i}$ (b)

03 $f(i\sqrt{3}) = 1-i$; $f(-i\sqrt{3}) = 2-i$; $g(z) = \frac{f(z)}{z^2+3}$

$\int_{|z|=2} g(z)\,dz = 2\pi i\left[\mathrm{Res}(g; i\sqrt{3}) + \mathrm{Res}(g; -i\sqrt{3})\right] =$

$$= 2\pi i\left[\frac{f(i\sqrt{3})}{2i\sqrt{3}} + \frac{f(-i\sqrt{3})}{-2i\sqrt{3}}\right] = 2\pi i\left(\frac{1-i}{2i\sqrt{3}} + \frac{2-i}{-2i\sqrt{3}}\right)$$

$$= 2\pi i\left(\frac{1-i-2+i}{2i\sqrt{3}}\right) = \boxed{-\frac{\pi}{\sqrt{3}}} \text{ (c)}$$

$\boxed{04}$ $z_0 = i$ pólo simples de $f(z)$

$$\lim_{z\to z_0} \frac{|1-z|}{|f(z)|} = \lim_{z\to z_0}\left(|1-z| \cdot \frac{|z-z_0|^2}{|(z-z_0)^2 f(z)|}\right) = \boxed{0} \text{ (a)}$$

$\boxed{05}$ $f(z) = z^{-1}\operatorname{Log}(1+z) = \frac{1}{z}\sum_{k=0}^{\infty}\frac{(-1)^k z^{k+1}}{k+1} = \sum_{k=0}^{\infty}\frac{(-1)^k z^k}{k+1}$

$z=0$ é uma sing. removível de $f(z)$ e, portanto, $\boxed{\operatorname{Res}(f;0)=0}$ (c)

$\boxed{06}$ $f(z) = \frac{-1}{(z-z_0)^2} + \frac{1}{i\pi\sqrt{2}(z-z_0)} + \sum_{k=0}^{\infty} a_k (z-z_0)^k$

$\operatorname{Res}(f;z_0) = -\frac{i}{\pi\sqrt{2}}$ e $\operatorname{Res}[(z-z_0)f(z);z_0] = -1$.

Logo:

$$\int_{|z-z_0|=1} f(z)dz + \int_{|z-z_0|=2}(z-z_0)f(z)\,dz = 2\pi i\left[-\frac{i}{\pi\sqrt{2}} - 1\right]$$

$$= \frac{2}{\sqrt{2}} - 2\pi i = \boxed{\sqrt{2} - 2\pi i} \text{ (d)}$$

$\boxed{07}$ $f(z) = \frac{1}{(z-i)^8}\frac{1}{i+(z-i)} = -i\sum_{k=0}^{\infty}\frac{(-1)^k (z-i)^{k-8}}{i^k}$

$(k=7)$ $\operatorname{Res}(f;i) = \frac{(-i)(-1)^7}{i^7} = \frac{i}{-i} = \boxed{-1}$ (c)

$\boxed{08}$ $\operatorname{Res}(f;-2) = \lim_{z\to -2}\frac{1}{z^2(z+4)} = \boxed{\frac{1}{8}}$

$\operatorname{Res}(f;-4) = \lim_{z\to -4}\frac{1}{z^2(z+2)} = \frac{-1}{32}$

Logo:

$$\int_\gamma f(z)dz = 2\pi i\left[\frac{1}{8} - \frac{1}{32}\right] = 2\pi i\left(\frac{3}{32}\right) = \boxed{\frac{3\pi i}{16}} \text{ (b)}$$

$\boxed{09}$ $f(z) = \frac{3z-2}{z(z^2+2)}$; $\operatorname{Res}(f,0) = \lim_{z\to 0}\frac{3z-2}{z^2+2} = \boxed{-1}$

$$\int_{|z|=1} f(z)dz = 2\pi i(-1) = \boxed{-2\pi i} \text{ (b)}$$

$\boxed{10}$ $\lim_{z\to 0} z^2 f(z) = \lim_{z\to 0}\frac{\operatorname{sen} z}{z} = 1$, $f(z) = z^{-3}\operatorname{sen} z$

(d)