AV 2 - INTEGRAÇÃO COMPLEXA VARIÁVEL COMPLEXA - 2017.2

ALUNO(A): MPMATOS ID - UFPB: ______

QUESTÕES MÚLTIPLA ESCOLHA COM RESPOSTA ÚNICA (valor 10 pontos) Nota: ______

01 *Sabendo que* $z = \frac{1}{\sqrt{2}}(-1-i)$, *assinale no menu o valor principal de* z^{2+i}.

(a) $\exp(\pi/2)$ (b) $-i\cdot\exp(3\pi)$ (c) $-\exp(-\pi)$ (d) $i\cdot\exp(3\pi/4)$ (e) $i\cdot\exp(-\pi/4)$ (f) NDR

02 *Escolha no menu o valor principal de* $\sqrt{4i}$

(a) $2\pi i$ (b) $\sqrt{2}+i\sqrt{2}$ (c)) $\sqrt{2}-i\sqrt{2}$ (d) $\ln 2+\pi i/2$ (e) $-\sqrt{2}+i\sqrt{2}$ (f) NDR

03 *Sobre o arco* $\gamma: z = e^{it}$, $\pi/4 \le t \le \pi/2$, *o valor da integral* $\int_\gamma (z^2\cdot\bar{z})\,dz$ *é:*

(a) $\frac{1}{2}(1-i)$ (b) $\frac{1}{2}(1+i)$ (c) $1+i$ (d) 0 (e) $i\sqrt{2}$ (f) NDR

04 *Se* γ *é a fronteira da região* $|x|+|y|\le 2$, *assinale o valor da integral* $\frac{1}{2\pi i}\int_\gamma \frac{z^2+2iz}{z-i}dz$.

(a) -3 (b) $2i$ (c) $-3i$ (d) 0 (e) $-1+2i$ (f) NDR

05 *Se* γ *é o arco* $|z|=1$, $\operatorname{Im}(z)\ge 0$, $\operatorname{Re}(z)\ge 0$, *o valor da integral* $\int_\gamma (iz)^3\,dz$ *é igual a:*

(a) -3 (b) 0 (c) $2i$ (d) $4i$ (e) $-i$ (f) NDR

06 *Se* γ *é o contorno* $|z-2i|=2$, *assinale o valor de* $\int_\gamma \frac{z^3}{z^2+4}dz$.

(a) 3π (b) $-2\pi i$ (c) $-4\pi i$ (d) $3\pi i$ (e) $2\pi i$ (f) NDR

07 *Certa função inteira* $f(z)$ *é tal que* $f(z) = (z-i)^2\operatorname{Log} z$, *nos pontos do contorno* $\gamma: |z-i| = 1/2$. *Assinale o valor de* $f''(i)$.

(a) $-\pi i/2$ (b) $4\pi i$ (c) 0 (d) $-2\pi i$ (e) πi (f) NDR

08 *Seja* γ *um arco de circunferência ligando os pontos* $z=0$ e $z=\pi i/2$. *Assinale o valor de* $\int_\gamma e^z dz$.

(a) $\sqrt{2}$ (b) 0 (c) -2 (d) $-1-i$ (e) $-1+i$ (f) NDR

09 *O segmento de reta* γ *liga o ponto* $z=1$ *ao ponto* $z=-1+i$. *Assinale o valor de* $\int_\gamma \frac{dz}{z}$.

(a) $\ln\sqrt{2}+\pi i/4$ (b) $-i\pi/2$ (c) $\ln\sqrt{2}+3\pi i/4$ (d) $\ln 2+i\pi/4$ (e) $\ln 2+i\pi/2$ (f) NDR

10 *Se* n *representa um número inteiro, assinale as raizes da equação* $\exp(2iz) = -i$.

(a) $n\pi+\pi/2$ (b) $n\pi+\pi/8$ (c) $n\pi+3\pi/8$ (d) $n\pi-\pi/4$ (e) $n\pi+\pi/4$ (f) NDR

GABARITO (PREENCHIMENTO OBRIGATÓRIO)

01	02	03	04	05	06	07	08	09	10
(a)	(a)	(a)	~~(a)~~	(a)	(a)	(a)	(a)	(a)	(a)
(b)	~~(b)~~	(b)	(b)	~~(b)~~	(b)	(b)	(b)	(b)	(b)
(c)	(c)	(c)	(c)	(c)	~~(c)~~	(c)	(c)	~~(c)~~	(c)
~~(d)~~	(d)	(d)	(d)	(d)	(d)	(d)	(d)	(d)	~~(d)~~
(e)	(e)	(e)	(e)	(e)	(e)	~~(e)~~	~~(e)~~	(e)	(e)
(f)	(f)	~~(f)~~	(f)	(f)	(f)	(f)	(f)	(f)	(f)

01 $z = \frac{1}{\sqrt{2}}(-1-i) = \exp\left(-\frac{3\pi i}{4}\right)$

$z^{2+i} = \exp\left[(2+i)\operatorname{Log}\left(e^{-\frac{3\pi i}{4}}\right)\right] = \exp\left[(2+i)\left(-i\frac{3\pi}{4}\right)\right]$

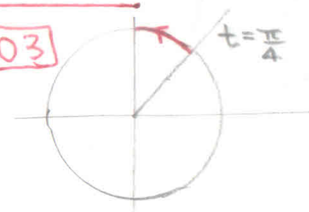
$= \exp\left[\frac{3\pi}{4} - i\frac{3\pi}{2}\right] = \boxed{i\,e^{3\pi/4}}$ (d)

02 $VP(\sqrt{4i}) = \exp\left[\frac{1}{2}\operatorname{Log}(4i)\right] = \exp\left[\frac{1}{2}\left(\ln 4 + i\frac{\pi}{2}\right)\right]$

$= \exp\left(\ln 2 + i\frac{\pi}{4}\right) = 2\cdot\left(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right)$

$= \boxed{\sqrt{2} + i\sqrt{2}}$ (b)

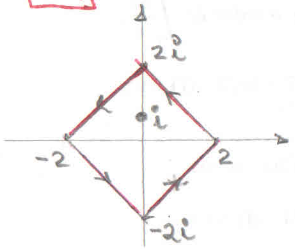
03



$\int_{\pi/4}^{\pi/2} i\,e^{2it}\,dt = i\left[\frac{e^{2it}}{2i}\right]_{\pi/4}^{\pi/2}$

$= \boxed{-\frac{1}{2}(1+i)}$ (f)

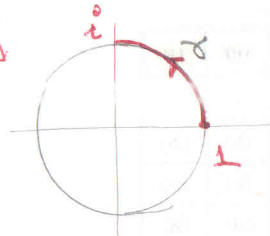
04



$$\frac{1}{2\pi i}\int_\gamma \frac{z^2+2iz}{z-i} = \frac{1}{2\pi i}\int_\gamma \frac{f(z)\,dz}{z-i} = f(i)$$

$$= (i)^2 + 2i(i) = \boxed{-3} \quad (a)$$

05



$$\int_\gamma (iz)^3\,dz = i\left[\frac{z^4}{4}\right]_1^i = \frac{i}{4}(1^4-1)$$

$$= \boxed{0} \quad (b)$$

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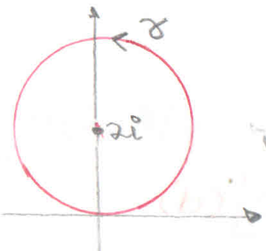
06 $\gamma : |z-2i| = 2$



$$\int_\gamma \frac{z^3\,dz}{z^2+4} = \int \frac{f(z)\,dz}{z-2i} \ ; \ f = \frac{z^3}{z+2i}$$

Logo:

$$\int_\gamma \frac{z^3\,dz}{z^2+4} = 2\pi i\, f(2i) = 2\pi i\left(\frac{-8i}{4i}\right)$$

$$= \boxed{-4\pi i} \quad (c)$$

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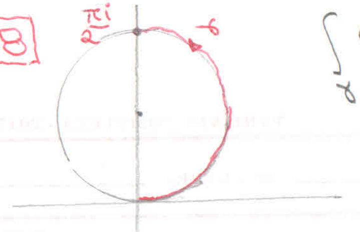
07 Da fórmula de Cauchy, temos:

$$f''(i) = \frac{2!}{2\pi i}\int_\gamma \frac{f(z)\,dz}{(z-i)^3} = \frac{1}{\pi i}\int_\gamma \frac{\operatorname{Log} z\,dz}{z-i}$$

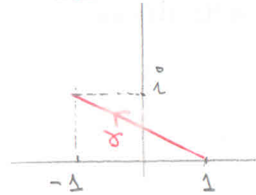
$$= \frac{1}{\pi i}\left(2\pi i \operatorname{Log} i\right) = 2\left(i\frac{\pi}{2}\right) = \boxed{\pi i} \quad (e)$$

08



$$\int_\gamma e^z\,dz = e^{\frac{\pi i}{2}} - e^0 = \boxed{i-1} \quad (e)$$

09



$$\int_\gamma \frac{dz}{z} = \left[\operatorname{Log} z\right]_1^{-1+i} = \operatorname{Log}(-1+i)$$

$$= \operatorname{Log}\left(\sqrt{2}\, e^{\frac{3\pi i}{4}}\right) = \boxed{\ln\sqrt{2} + \frac{3\pi i}{4}} \quad (c)$$

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10 $\exp(2iz) = -i \Leftrightarrow 2iz = \operatorname{Log}(-i) + 2n\pi i$

$$\Leftrightarrow 2iz = -i\frac{\pi}{2} + 2n\pi i$$

$$\Leftrightarrow \boxed{z = n\pi - \frac{\pi}{4}}$$

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