



ALUNO(A): MPMATOS

ID - UFPB: _____

QUESTÕES MÚLTIPLA ESCOLHA COM RESPOSTA ÚNICA (valor 10 pontos)

Nota:

01 Sabendo que $z = 1 + i^{15}$ e $w = 1 + i + i^2 + \dots + i^{14}$, escolha no menu o valor de $\overline{z \cdot w}$.

- (a) $1 + i$ (b) $-1 + i$ (c) $-1 - i$ (d) $1 - i$ (e) 1 (f) **NDR**

02 No domínio $D : r > 0, 0 < \theta < 2\pi$, seja $F(re^{i\theta}) = \sqrt{r}e^{i\theta/2}$. Assinale o valor de $4 \cdot F'(-i)$.

- (a) $-\sqrt{2} - i\sqrt{2}$ (b) $\sqrt{2} + i\sqrt{2}$ (c) $-\sqrt{2} + i\sqrt{2}$ (d) $\sqrt{2} - i\sqrt{2}$ (e) $-i\sqrt{2}$ (f) **NDR**

03 Seja $P(z) = a_0 + a_1z + a_2z^2 + a_3z^3$ com $a_3 \neq 0$, um polinômio com coeficientes reais. Assinale o valor de $i \cdot P(\sqrt{2} + i)$, sabendo que $P(\sqrt{2} - i) = 1 + i\sqrt{2}$.

- (a) $3 - i$ (b) $-2 - i\sqrt{2}$ (c) $\sqrt{2} + i$ (d) $2 + 2i$ (e) $1 + i\sqrt{2}$ (f) **NDR**

04 Assinale a função $f : \mathbb{C} \rightarrow \mathbb{C}$ que é derivável em todos os pontos do plano \mathbb{C} .

- (a) $f(z) = \text{Re}(z)$ (b) $f(z) = \bar{z}$ (c) $f(z) = z \cdot \bar{z}$ (d) $f(z) = e^x e^{iy}$ (e) $f(z) = 1/z$ (f) **NDR**

05 Escolha no menu o valor da expressão $|\sqrt[3]{2 + i\sqrt{5}}|$.

- (a) $\sqrt[3]{3}$ (b) $\sqrt[3]{4}$ (c) 2 (d) $\sqrt[3]{5}$ (e) $\sqrt[3]{6}$ (f) **NDR**

06 Se $|z| = \sqrt{2}$, então o valor de $|1 - (2 + i)z|^2 + |1 + (2 + i)z|^2$ é igual a:

- (a) 4 (b) 18 (c) 14 (d) 22 (e) 10 (f) **NDR**

07 Se $f(z) = -x^2 + y^2 + 2xy + i(-2xy - x^2 + y^2)$, assinale o valor de $f'(1 - i)$.

- (a) -4 (b) 4 (c) $4 - 8i$ (d) $-4i$ (e) $-8i$ (f) **NDR**

08 Seja $g(z)$ uma função derivável em uma vizinhança de $z = i$ e suponha que $g(i) = 0$ e $g'(i) = 4 + i$.

Assinale o valor de:

$$\lim_{z \rightarrow i} \frac{g(z)}{1 + z^2}$$

- (a) $\frac{1}{2} + 2i$ (b) $\frac{1}{2} + i$ (c) $\frac{1}{2} - 2i$ (d) $\frac{1}{2} - i$ (e) $\frac{1}{2} + 4i$ (f) **NDR**

09 Se $f(z) = \frac{-1-i}{iz}$, $z \neq 0$, assinale no menu o valor do limite: $\lim_{z \rightarrow 2i} \frac{f(z) - f(2i)}{z - 2i}$.

- (a) $\frac{1}{4}(-1 - i)$ (b) $2i$ (c) $-1 + i$ (d) $\frac{1}{2}(1 + i)$ (e) $\frac{1}{4}(-1 + i)$ (f) **NDR**

10 Seja $f(z) = u(x, y) + iv(x, y)$, sendo $u(x, y) = 4(x^2 - y^2) + 2xy$ e $v(x, y)$ a harmônica conjugada de $u(x, y)$. Assinale o valor de $f(\sqrt{2} + i\sqrt{2})$.

- (a) $-8 - 32i$ (b) $8 + 32i$ (c) $4 + 16i$ (d) $-4 + 16i$ (e) $2 + 8i$ (f) **NDR**

GABARITO (PREENCHIMENTO OBRIGATÓRIO)

01	02	03	04	05	06	07	08	09	10
(a)	(a)	(a)	(a)	(a)	(a)	(a)	(a)	(a)	(a)
(b)	(b)	(b)	(b)	(b)	(b)	(b)	(b)	(b)	(b)
(c)	(c)	(c)	(c)	(c)	(c)	(c)	(c)	(c)	(c)
(d)	(d)	(d)	(d)	(d)	(d)	(d)	(d)	(d)	(d)
(e)	(e)	(e)	(e)	(e)	(e)	(e)	(e)	(e)	(e)
(f)	(f)	(f)	(f)	(f)	(f)	(f)	(f)	(f)	(f)

(01) $z = 1 - i, w = \frac{1+i}{1-i} \Rightarrow \overline{z \cdot w} = \overline{z} \cdot \overline{w} = (1+i) \cdot \left(\frac{1+i}{1-i}\right)$

(02) $F(-i) = \frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2} \Rightarrow 4F'(-i) = \frac{4}{2F(i)} = \frac{-1+i}{\sqrt{2}+i\sqrt{2}}$ (b)

(03) $i \cdot P(\sqrt{2} + i) = i(1 - i\sqrt{2}) = \sqrt{2} + i$ (c)

(04) $f(z) = e^z = e^x \cdot e^{iy}$

(05) $|z| = \sqrt[3]{3}$ (a)

(06) $2[1^2 + |2+i|^2] = 2(1 + 5) = 12$ (d)

(07) $f'(1-i) = u_x(1,-1) + i u_y(1,-1) = -4$ (a)

$$(08) \lim_{z \rightarrow i} \frac{f(z)}{1+z^2} = \frac{f'(i)}{2i} = \frac{4+i}{2i} = \boxed{\frac{1}{2} - 2i} \quad (c)$$

$$(09) f'(z) = \frac{(1+i)i}{-z^2} \Rightarrow f'(2i) = \frac{-1+i}{4} \quad (e)$$

$$(10) f(\sqrt{2} + i\sqrt{2}) = u(\sqrt{2}, \sqrt{2}) + i v(\sqrt{2}, \sqrt{2}) \\ = \boxed{4 + 16i} \quad (c)$$

$$v_x = -u_y = 8y - 2x \Rightarrow v = 8xy - x^2 + K(y)$$

$$v_y = u_x \Rightarrow 8x + K'(y) = 8x + 2y \Rightarrow K = y^2$$

$$v = -x^2 + y^2 + 8xy$$