



7.1 Mudança de Variável (Método de Substituição)

1. $\int e^{5x} dx$ (resp. $\frac{1}{5} \exp(5x) + C$)
2. $\int \operatorname{sen}(ax) dx$ (resp. $-\frac{1}{a} \cos(ax) + C, a \neq 0$)
3. $\int \frac{dx}{\operatorname{sen}^2(3x)}$ (resp. $-\frac{1}{3} \operatorname{cotg}(3x) + C$)
4. $\int \frac{dx}{3x-7}$ (resp. $\frac{1}{3} \ln |3x-7| + C$)
5. $\int \tan(2x) dx$ (resp. $-\frac{1}{2} \ln |\cos(2x)| + C$)
6. $\int \operatorname{cotg}(5x-7) dx$ (resp. $\frac{1}{5} \ln |\operatorname{sen}(5x-7)| + C$)
7. $\int \tan x \sec^2 x dx$ (resp. $\frac{1}{2} \tan^2 x + C$)
8. $\int (\cot e^x) e^x dx$ (resp. $\ln |\operatorname{sen}(e^x)| + C$)
9. $\int \operatorname{sen}^2 x \cos x dx$ (resp. $\frac{1}{3} \operatorname{sen}^3 x + C$)
10. $\int \cos^3 x \operatorname{sen} x dx$ (resp. $-\frac{1}{4} \cos^4 x + C$)
11. $\int x\sqrt{x^2+1} dx$ (resp. $\frac{1}{3}(x^2+1)^{3/2} + C$)
12. $\int \frac{x^2 dx}{\sqrt{x^3+1}}$ (resp. $\frac{2}{3}\sqrt{x^3+1} + C$)
13. $\int \frac{\cos x dx}{\operatorname{sen}^2 x}$ (resp. $-\operatorname{cosec} x + C$)
14. $\int \frac{dx}{\cos^2 x \sqrt{\tan x - 1}}$ (resp. $2\sqrt{\operatorname{tg} x - 1} + C$)
15. $\int \frac{\ln(x+1) dx}{x+1}$ (resp. $\frac{1}{2} [\ln(x+1)]^2 + C$)

16. $\int \frac{\cos x dx}{\sqrt{2 \operatorname{sen} x + 1}}$ (resp. $\sqrt{1 + 2 \operatorname{sen} x} + C$)
17. $\int \frac{\sqrt{\tan x + 1}}{\cos^2 x} dx$ (resp. $\frac{2}{3}(1 + \operatorname{tg} x)^{3/2} + C$)
18. $\int \frac{\cos 2x dx}{(2 + 3 \operatorname{sen} 2x)^3}$ (resp. $-\frac{1}{12}(2 + 3 \operatorname{sen} 2x)^{-2} + C$)
19. $\int \frac{\arcsin x dx}{\sqrt{1 - x^2}}$ (resp. $\frac{1}{2} \operatorname{arcsen}^2 x + C$)
20. $\int \frac{\arctan x dx}{1 + x^2}$ (resp. $\frac{1}{2} \operatorname{arctg}^2 x + C$)
21. $\int \frac{\arccos^2 x dx}{\sqrt{1 - x^2}}$ (resp. $-\frac{1}{3} \arccos^3 x + C$)
22. $\int \frac{(x + 1) dx}{x^2 + 2x + 3}$ (resp. $\frac{1}{2} \ln(x^2 + 2x + 3) + C$)
23. $\int \frac{\cos x dx}{2 \operatorname{sen} x + 3}$ (resp. $\frac{1}{2} \ln(3 + 2 \operatorname{sen} x) + C$)
24. $\int \frac{dx}{x \ln x}$ (resp. $\ln |\ln x| + C$)
25. $\int \frac{dx}{(1 + x^2) \arctan x}$ (resp. $\ln |\operatorname{arctg} x| + C$)
26. $\int \frac{\tan^3 x}{\cos^2 x} dx$ (resp. $\frac{1}{4} \operatorname{tg}^4 x + C$)
27. $\int \frac{dx}{\sqrt{1 - x^2} \arcsin x}$ (resp. $\ln |\operatorname{arcsen} x| + C$)
28. $\int \frac{\cos(\ln x)}{x} dx$ (resp. $\operatorname{sen}(\ln |x|) + C$)
29. $\int x a^{x^2} dx, \quad a > 0$ (resp. $\frac{a^{x^2}}{2 \ln a} + C$)
30. $\int 3^x e^x dx$ (resp. $\frac{(3e)^x}{1 + \ln 3} + C$)
31. $\int e^{-3x} dx$ (resp. $-\frac{e^{-3x}}{3} + C$)
32. $\int (e^{5x} + a^{5x}) dx, \quad a > 0$ (resp. $-\frac{e^{5x}}{5} + \frac{a^{5x}}{5 \ln a} + C$)
33. $\int e^{x^2 + 4x + 3} (x + 2) dx$ (resp. $\frac{1}{2} \exp(x^2 + 4x + 3) + C$)

34. $\int \frac{(a^x - b^x)^2}{a^x b^x} dx, \quad a > b > 0 \dots\dots\dots$ (resp. $-2x + \frac{(a/b)^x - (b/a)^x}{\ln(a/b)} + C$)
35. $\int \frac{\exp(2x) dx}{2 + \exp(2x)} \dots\dots\dots$ (resp. $\frac{1}{2} \ln(2 + e^{2x}) + C$)
36. $\int \frac{dx}{\sqrt{1 - 3x^2}} \dots\dots\dots$ (resp. $\frac{\sqrt{3}}{3} \arcsen(\sqrt{3}x) + C$)
37. $\int \frac{dx}{\sqrt{16 - 9x^2}} \dots\dots\dots$ (resp. $\frac{1}{3} \arcsen(3x/4) + C$)
38. $\int \frac{dx}{9x^2 + 4} \dots\dots\dots$ (resp. $\frac{1}{6} \operatorname{arctg}(3x/2) + C$)
39. $\int \frac{x dx}{\sqrt{1 - x^4}} \dots\dots\dots$ (resp. $\frac{1}{2} \arcsen(x^2) + C$)
40. $\int \frac{x dx}{x^4 + a^4}, \quad a > 0 \dots\dots\dots$ (resp. $\frac{1}{2a^2} \operatorname{arctg}(x^2/a^2) + C$)
41. $\int \frac{e^x dx}{\sqrt{1 - e^{2x}}} \dots\dots\dots$ (resp. $\arcsen(e^x) + C$)
42. $\int \frac{\cos x dx}{a^2 + \operatorname{sen}^2 x}, \quad a > 0 \dots\dots\dots$ (resp. $\frac{1}{a} \operatorname{arctg}[(\operatorname{sen} x)/a] + C$)
43. $\int \frac{dx}{x \sqrt{1 - \ln^2 x}}, \quad 0 < x < e \dots\dots\dots$ (resp. $\arcsen(\ln x) + C$)
44. $\int \frac{(x - \operatorname{arctan} x) dx}{1 + x^2} \dots\dots\dots$ (resp. $\frac{1}{2} \ln(1 + x^2) - \frac{1}{2} \operatorname{arctg}^2 x + C$)
45. $\int \frac{\sqrt{1 + \sqrt{x}} dx}{\sqrt{x}} \dots\dots\dots$ (resp. $\frac{4}{3} (1 + \sqrt{x})^{3/2} + C$)
46. $\int \frac{dx}{\sqrt{x} \sqrt{1 + \sqrt{x}}} \dots\dots\dots$ (resp. $4(1 + \sqrt{x})^{1/2} + C$)
47. $\int \frac{e^x dx}{1 + e^{2x}} \dots\dots\dots$ (resp. $\operatorname{arctg}(e^x) + C$)
48. $\int \frac{\cos x dx}{\sqrt[3]{\operatorname{sen}^2 x}} \dots\dots\dots$ (resp. $3\sqrt{\operatorname{sen} x} + C$)
49. $\int \frac{\operatorname{sen} 2x dx}{\sqrt{1 + \cos^2 x}} \dots\dots\dots$ (resp. $-2\sqrt{1 + \cos^2 x} + C$)
50. $\int \frac{\cos^3 x dx}{\operatorname{sen}^4 x} \dots\dots\dots$ (resp. $(\operatorname{sen} x)^{-1} - \frac{1}{3}(\operatorname{sen} x)^{-3} + C$)
51. $\int \frac{\sqrt[3]{\tan^2 x} dx}{\cos^2 x} \dots\dots\dots$ (resp. $\frac{3}{5}(\operatorname{tg} x)^{5/3} + C$)

$$52. \int \frac{dx}{2 \operatorname{sen}^2 x + 3 \cos^2 x} \dots\dots\dots (\text{resp. } \frac{1}{\sqrt{6}} \operatorname{arctg}(\sqrt{\frac{2}{3}} \operatorname{tg} x) + C)$$

7.2 Integração por Partes

$$1. \int x e^x dx \dots\dots\dots (\text{resp. } x e^x - e^x + C)$$

$$2. \int x \ln x dx \dots\dots\dots (\text{resp. } \frac{1}{2} x^2 \ln x - \frac{1}{4} x^2 + C)$$

$$3. \int x \operatorname{sen} x dx \dots\dots\dots (\text{resp. } \operatorname{sen} x - x \cos x + C)$$

$$4. \int x \operatorname{sen}(kx) dx, \quad k = 1, 2, 3, \dots\dots\dots (\text{resp. } \frac{\operatorname{sen}(kx)}{k^2} - \frac{x \cos(kx)}{k} + C)$$

$$5. \int \ln x dx \dots\dots\dots (\text{resp. } x \ln x - x + C)$$

$$6. \int \operatorname{arcsen} x dx \dots\dots\dots (\text{resp. } x \operatorname{arcsen} x + \sqrt{1-x^2} + C)$$

$$7. \int x^n \ln x dx \dots\dots\dots (\text{resp. } \frac{x^{n+1}}{n+1} (\ln x - \frac{1}{n+1}) + C)$$

$$8. \int x \operatorname{arctan} x dx \dots\dots\dots (\text{resp. } \frac{1}{2} (x^2 + 1) \operatorname{arctg} x - \frac{1}{2} x + C)$$

$$9. \int \ln(x^2 + 1) dx \dots\dots\dots (\text{resp. } x \ln(1 + x^2) - 2x + 2 \operatorname{arctg} x + C)$$

$$10. \int \frac{\operatorname{arcsen} \sqrt{x}}{\sqrt{x}} dx \dots\dots\dots (\text{resp. } 2\sqrt{x} \operatorname{arcsen} \sqrt{x} + 2\sqrt{1-x} + C)$$

$$11. \int \operatorname{arcsen} \left(\sqrt{\frac{x}{x+1}} \right) dx \dots\dots\dots (\text{resp. } x \operatorname{arcsen} \sqrt{\frac{x}{x+1}} - \sqrt{x} + \operatorname{arctg} \sqrt{x} + C)$$

$$12. \int \frac{x \operatorname{arctan} x dx}{(x^2 + 1)^2} \dots\dots\dots (\text{resp. } \frac{x - (1-x^2) \operatorname{arctg} x}{4(1+x^2)} + C)$$

$$13. \int x \operatorname{arctan} (\sqrt{x^2 - 1}) dx \dots\dots\dots (\text{resp. } \frac{1}{2} [x^2 \operatorname{arctg} \sqrt{x^2 - 1} - \sqrt{x^2 - 1}] + C)$$

$$14. \int \frac{\operatorname{arcsen} x dx}{x^2} \dots\dots\dots (\text{resp. } \ln \left| \frac{1 - \sqrt{1-x^2}}{x} \right| - \frac{\operatorname{arcsen} x}{x} + C)$$

$$15. \int \ln (x + \sqrt{1+x^2}) dx \dots\dots\dots (\text{resp. } x \ln (x + \sqrt{1+x^2}) - \sqrt{1+x^2} + C)$$

- 16. $\int x^2 e^x dx$ (resp. $x^2 e^x - 2x e^2 + 2e^x + C$)
- 17. $\int_0^\pi x^2 \cos(kx) dx, \quad k = 1, 2, 3, \dots$ (resp. $\frac{2\pi(-1)^k}{k^2}$)
- 18. $\int_0^{2\pi} x^2 \text{sen}(kx) dx, \quad k = 1, 2, 3, \dots$ (resp. $\frac{-4\pi^2}{k}$)

7.3 Problema de Valor Inicial (PVI)

- 1. $\frac{ds}{dt} = 12t(3t^2 - 1)^3, \quad s(1) = 3$ (resp. $s(t) = \frac{1}{2}(3t^2 - 1)^4 - 5$)
- 2. $\frac{d^2y}{dx^2} = \sec^2 x, \quad y(0) = 1, \quad y'(0) = 0$ (resp. $y(t) = -\ln(1 + |\cos t|)$)
- 3. $\frac{dy}{dt} = e^t \text{sen}(e^t - 2), \quad y(\ln 2) = 0$ (resp. $y(t) = -\cos(e^t - 2) + 1$)

7.4 Decomposição em Frações Parciais

- 1. $\int \frac{(2x - 1)}{(x - 1)(x - 2)} dx$ (resp. $-\ln|x - 1| + 3\ln|x - 2| + C$)
- 2. $\int \frac{xdx}{(x + 1)(x + 3)(x + 5)}$ (resp. $-\frac{1}{8}\ln|x + 1| + \frac{3}{4}\ln|x + 3| - \frac{5}{8}\ln|x + 5| + C$)
- 3. $\int \frac{(x^5 + x^4 - 8) dx}{x^3 - 4x}$ (resp. $4x + 2\ln|x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + 5\ln|x - 2| - 3\ln|x + 2| + C$)
- 4. $\int \frac{x^4 dx}{(x^2 - 1)(x + 2)}$ (resp. $\frac{1}{2}x^2 - 2x + \frac{1}{2}\ln\left|\frac{\sqrt[3]{x-1}}{x+1}\right| + \frac{16}{3}\ln|x + 2| + C$)
- 5. $\int \frac{dx}{(x - 1)^2(x - 2)}$ (resp. $\frac{1}{x - 1} + \ln\left|\frac{x - 2}{x - 1}\right| + C$)
- 6. $\int \frac{(x - 8) dx}{x^3 - 4x^2 + 4x}$ (resp. $-2\ln|x| + \frac{3}{x - 2} + 2\ln|x - 2| + C$)
- 7. $\int \frac{(3x + 2) dx}{x(x + 1)^3}$ (resp. $2\ln|x| - \frac{1}{2(x + 1)^2} + \frac{2}{x + 1} - 2\ln|x + 1| + C$)
- 8. $\int \frac{dx}{x^3 + 1}$ (resp. $\frac{1}{3}\ln\left|\frac{x + 1}{\sqrt{x^2 - x + 1}}\right| + \frac{1}{\sqrt{3}}\text{arctg}\left(\frac{2x - 1}{\sqrt{3}}\right) + C$)

9. $\int \frac{x^5}{x^3 - 1} \dots\dots\dots$ (resp. $\frac{1}{3}x^3 + \frac{1}{3} \ln |x^3 - 1| + C$)
10. $\int \frac{(x^2 + 2x + 1) dx}{(1 + x^2)^2} \dots\dots\dots$ (resp. $-\frac{1}{1 + x^2} + \operatorname{arctg} x + C$)
11. $\int \frac{dx}{x^2 + 2x + 5} \dots\dots\dots$ (resp. $\frac{1}{2} \operatorname{arctg} \left[\frac{1}{2}(x + 1) \right] + C$)
12. $\int \frac{dx}{3x^2 - 2x + 4} \dots\dots\dots$ (resp. $\frac{1}{\sqrt{11}} \operatorname{arctg} \left(\frac{3x - 1}{\sqrt{11}} \right) + C$)
13. $\int \frac{dx}{x^2 - 6x + 5} \dots\dots\dots$ (resp. $\frac{1}{4} \ln \left| \frac{x - 5}{x - 1} \right| + C$)
14. $\int \frac{dx}{2x^2 - 2x + 1} \dots\dots\dots$ (resp. $\operatorname{arctg} (2x - 1) + C$)
15. $\int \frac{(6x - 7) dx}{3x^2 - 7x + 11} \dots\dots\dots$ (resp. $\ln |3x^2 - 7x + 11| + C$)
16. $\int \frac{(7x + 1) dx}{6x^2 + x - 1} \dots\dots\dots$ (resp. $\frac{1}{2} \ln |2x + 1| + \frac{2}{3} \ln |3x - 1| + C$)
17. $\int \frac{(3x - 2) dx}{5x^2 - 3x + 2} \dots\dots\dots$ (resp. $\frac{3}{10} \ln |5x^2 - 3x + 2| - \frac{11}{5\sqrt{31}} \operatorname{arctg} \left(\frac{10x - 3}{\sqrt{31}} \right) + C$)
18. $\int \frac{(6x^4 - 5x^3 + 4x^2) dx}{2x^2 - x + 1} \dots\dots\dots$ (resp. $x^3 - \frac{1}{2}x^2 + \frac{1}{4} \ln |2x^2 - x + 1| + \frac{1}{2\sqrt{7}} \operatorname{arctg} \left(\frac{4x - 1}{\sqrt{7}} \right) + C$)

7.5 Integrais Trigonométricas

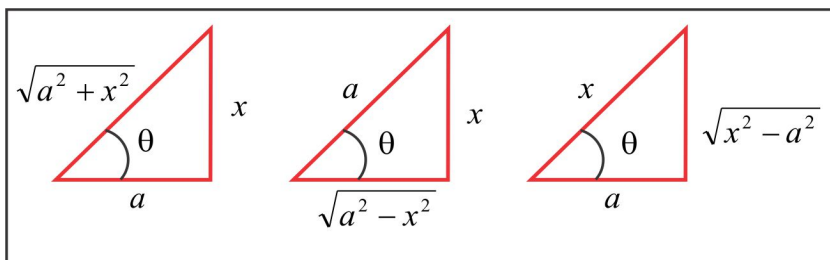
1. $\int \operatorname{sen}^3 x dx \dots\dots\dots$ (resp. $-\cos x + \frac{1}{3} \cos^3 x + C$)
2. $\int \operatorname{sen}^5 x dx \dots\dots\dots$ (resp. $-\cos x + \frac{2}{3} \cos^3 x - \frac{1}{5} \cos^5 x + C$)
3. $\int \cos^4 x \operatorname{sen}^3 x dx \dots\dots\dots$ (resp. $-\frac{1}{5} \cos^5 x + \frac{1}{7} \cos^7 x + C$)
4. $\int \frac{\cos^3 x dx}{\operatorname{sen}^4 x} \dots\dots\dots$ (resp. $\operatorname{cosec} x - \frac{1}{3} \operatorname{cosec}^3 x + C$)
5. $\int \tan^3 x dx \dots\dots\dots$ (resp. $\frac{1}{2} \tan^2 x + \log |\cos x| + C$)
6. $\int \cotg^5 x dx \dots\dots\dots$ (resp. $-\frac{1}{4} \operatorname{cosec}^4 x + \operatorname{cosec}^2 x + \log |\operatorname{sen} x| + C$)

- 7. $\int \cos^4 x dx$ (resp. $\frac{1}{4} \sin x \cos^3 x + \frac{3}{8} \sin x \cos x + \frac{3}{8} x + C$)
- 8. $\int \tan^2 x dx$ (resp. $\tan x - x + C$)
- 9. $\int \frac{\cos^{20} x}{\sin^{22} x} dx$ (resp. $-\frac{1}{21} \cot^2 x + C$)
- 10. $\int \cos 4x \cos 7x dx$ (resp. $\frac{1}{6} \sin(3x) + \frac{1}{22} \sin(11x) + C$)
- 11. $\int \cos^2 x \sin^3 x dx$ (resp. $\frac{1}{5} \cos^5 x - \frac{1}{3} \cos^3 x + C$)
- 12. $\int \sqrt[3]{\cos x} \sin^5 x dx$ (resp. $-\frac{3}{16} \cos^{16/3} x + \frac{3}{5} \cos^{10/3} x - \frac{3}{4} \cos^{4/3} x + C$)
- 13. $\int \sqrt{\sin x} \cos x dx$ (resp. $\frac{2}{3} \sin^{3/2} x + C$)
- 14. $\int \frac{\cot^3 x}{\sqrt{\sin x}} dx$ (resp. $-\frac{2}{5 \sin^{5/2} x} + \frac{2}{\sqrt{\sin x}} + C$)
- 15. $\int \sin^p x \cos^3 x dx$ (resp. $\frac{(\sin x)^{1+p}}{1+p} - \frac{(\sin x)^{3+p}}{3+p} + C$)
- 16. $\int \sin^2 x dx$ (resp. $-\frac{1}{2} \cos x \sin x + \frac{1}{2} x + C$)
- 17. $\int \sec x dx$ (resp. $\log |\sec x + \operatorname{tg} x| + C$)
- 18. $\int \sec^3 x dx$ (resp. $\frac{1}{2} \sin x \sec^2 x + \frac{1}{2} \log |\sec x + \operatorname{tg} x| + C$)
- 19. $\int \cos^2 x dx$ (resp. $\frac{1}{2} \cos x \sin x + \frac{1}{2} x + C$)
- 20. $\int \sin^2 x \sec^3 x dx$ (resp. $\frac{1}{2} \sin^3 x \sec^2 x + \frac{1}{2} \sin x - \frac{1}{2} \log |\sec x + \tan x| + C$)
- 21. $\int \sin^2 x \cos^2 x dx$ (resp. $-\frac{1}{4} \cos^3 x \sin x + \frac{1}{8} \cos x \sin x + \frac{1}{8} x + C$)
- 22. $\int \cos^6 x dx$ (resp. $(\frac{1}{6} \cos^4 x + \frac{5}{24} \cos^2 x + \frac{5}{16}) \cos x \sin x + \frac{5}{16} x + C$)
- 23. $\int \operatorname{cosec}^4 x dx$ (resp. $-\cot x - \frac{1}{3} \cot^3 x + C$)
- 24. $\int \tan^4 x \sec^4 x dx$ (resp. $\frac{1}{7} \tan^7 x + \frac{1}{5} \tan^5 x + C$)

- 25. $\int \cos 2x \operatorname{sen} 4x dx$ (resp. $-\frac{1}{12} \cos 6x - \frac{1}{4} \cos 2x + C$)
- 26. $\int \operatorname{sen}^4 x dx$ (resp. $-\frac{1}{2} \cos x \operatorname{sen} x + \frac{1}{2} x + C$)
- 27. $\int \sec^8 x dx$ (resp. $\frac{1}{7} \tan^7 x + \frac{3}{5} \tan^5 x + \tan^3 x + \tan x + C$)
- 28. $\int \operatorname{sen} x \operatorname{sen} 3x dx$ (resp. $\frac{1}{4} \sin 2x - \frac{1}{8} \sin 4x + C$)
- 29. $\int \frac{\cos^2 x}{\operatorname{sen}^4 x} dx$ (resp. $-\frac{1}{3} \cot^3 x + C$)
- 30. $\int \operatorname{sen}(x/4) \cos(3x/4) dx$ (resp. $-\frac{1}{2} \cos x + \cos(x/2) + C$)

■ INTEGRAIS ENVOLVENDO $\sqrt{x^2 \pm a^2}$ e $\sqrt{a^2 \pm x^2}$

As integrais que envolvem as expressões $\sqrt{x^2 \pm a^2}$ e $\sqrt{a^2 \pm x^2}$ podem ser calculadas com auxílio de substituições especiais. As substituições mais comuns são: $x = a \operatorname{tg} \theta$, $x = a \operatorname{sen} \theta$ e $x = a \operatorname{sec} \theta$ e elas são deduzidas a partir das relações nos triângulos retângulos da figura abaixo:



Substituições Trigonômétricas		
1. a mudança $x = a \operatorname{tg} \theta$	substitui $a^2 + x^2$	por $a^2 \sec^2 \theta$
2. a mudança $x = a \operatorname{sen} \theta$	substitui $a^2 - x^2$	por $a^2 \cos^2 \theta$
3. a mudança $x = a \operatorname{sec} \theta$	substitui $x^2 - a^2$	por $a^2 \operatorname{tg}^2 \theta$

- 1. $\int \frac{\sqrt{a^2 - x^2} dx}{x^2}$ (resp. $-\frac{\sqrt{a^2 - x^2}}{x} - 2 \operatorname{arcsen}(x/a) + C$)
- 2. $\int \frac{x^2}{\sqrt{4 - x^2}} dx$ (resp. $-\frac{x}{2} \sqrt{4 - x^2} + 2 \operatorname{arcsen}(x/2) + C$)
- 3. $\int \frac{dx}{\sqrt{(a^2 + x^2)^3}}$ (resp. $\frac{x}{a^2 \sqrt{a^2 + x^2}} + C$)

4. $\int \frac{x dx}{\sqrt{4x^2 + 8x + 5}}$ (resp. $\frac{1}{4}\sqrt{4x^2 + 8x + 5} - \frac{1}{2} \ln(2 + 2x + \sqrt{4x^2 + 8x + 5}) + C$)
5. $\int \frac{dx}{\sqrt{x^2 - 4x - 5}}$ (resp. $\ln(-2 + x + \sqrt{x^2 - 4x - 5}) + C$)
6. $\int \frac{dx}{\sqrt{x^2 + 2x + 2}}$ (resp. $\ln(1 + x + \sqrt{x^2 + 2x + 2}) + C$)
7. $\int \frac{x dx}{\sqrt{6x - x^2 - 5}}$ (resp. $-\sqrt{-x^2 + 6x - 5} + 3\arcsen(\frac{1}{2}x - \frac{3}{2}) + C$)
8. $\int \frac{dx}{\sqrt{1 + x^2}}$ (resp. $\ln(x + \sqrt{x^2 + 1}) + C$)
9. $\int \frac{dx}{\sqrt{x^2 - 4}}$ (resp. $\ln(x + \sqrt{x^2 + 4}) + C$)
10. $\int \frac{dx}{\sqrt{4x^2 - 4x - 3}}$ (resp. $\frac{1}{2} \ln(2x - 1 + \sqrt{4x^2 - 4x - 3}) + C$)
11. $\int x^2 \sqrt{4 - x^2} dx$ (resp. $-\frac{1}{4}x(4 - x^2)^{3/2} + \frac{1}{2}x\sqrt{4 - x^2} + 2 \arcsin(x/2) + C$)
12. $\int \frac{x dx}{\sqrt{8 - x^2 - 2x}}$ (resp. $-\sqrt{-x^2 - 2x + 8} - \arcsin \frac{1}{3}(x + 1) + C$)
13. $\int \frac{dx}{x^2 \sqrt{x^2 + 9}}$ (resp. $-\frac{\sqrt{x^2 + 9}}{9x} + C$)
14. $\int \frac{dx}{x^2 \sqrt{x^2 - 16}}$ (resp. $-\frac{\sqrt{x^2 - 16}}{16x} + C$)
15. $\int \frac{\sqrt{x^2 - a^2} dx}{x}$ (resp. $\sqrt{x^2 - a^2} - a \operatorname{arcsec}(x/a) + C$)
16. $\int \frac{dx}{\sqrt{2 - 3x - 4x^2}}$ (resp. $\frac{1}{2} \arcsin [(8x + 3)/\sqrt{41}] + C$)
17. $\int \frac{dx}{\sqrt{1 + x + x^2}}$ (resp. $\ln|x + \sqrt{x^2 + x + 1} + \frac{1}{2}| + C$)
18. $\int \frac{dx}{\sqrt{5 - 7x - 3x^2}}$ (resp. $\frac{1}{\sqrt{3}} \arcsin [(6x + 7)/\sqrt{109}] + C$)
19. $\int \frac{dx}{\sqrt{x(3x + 5)}}$ (resp. $\frac{1}{\sqrt{3}} \log|6x + 5 + \sqrt{36x^2 + 60x}| + C$)
20. $\int \frac{dx}{\sqrt{5x^2 - x - 1}}$ (resp. $\frac{1}{\sqrt{5}} \log|10x - 1 + 2\sqrt{5(5x^2 - x - 1)}| + C$)
21. $\int \frac{(x + 3) dx}{\sqrt{4x^2 + 4x + 3}}$ (resp. $\frac{1}{4}\sqrt{4x^2 + 4x + 3} + \frac{5}{4} \ln|1 + 2x + \sqrt{4x^2 + 4x + 3}| + C$)

$$22. \int \frac{(x-3) dx}{\sqrt{3+66x-11x^2}} \dots\dots\dots (\text{resp. } -\frac{1}{11}\sqrt{-11x^2+66x+3} + C)$$

$$23. \int \frac{(3x+5) dx}{\sqrt{x(2x-1)}} \dots\dots\dots (\text{resp. } \frac{3}{2}\sqrt{2x^2-x} + \frac{23}{4\sqrt{2}} \log|4x-1+\sqrt{16x^2-8x}| + C)$$

7.6 Integral de Funções Especiais

■ FUNÇÕES ENVOLVENDO $\exp x$

$$1. \int \tanh x dx \dots\dots\dots (\text{resp. } \log(\cosh x) + C)$$

$$2. \int \frac{e^x dx}{e^x - 1} \dots\dots\dots (\text{resp. } \log(e^x - 1) + C)$$

$$3. \int \frac{1 + \sinh x}{1 + \cosh x} dx \dots\dots\dots (\text{resp. } \tanh(x/2) - \log(-1 + \tanh(x/2)) - \log(1 + \tanh(x/2)) + C)$$

$$4. \int \frac{e^x}{e^{2x} + 1} dx \dots\dots\dots (\text{resp. } \arctan e^x + C)$$

■ FUNÇÕES ENVOLVENDO RADICAIS

$$1. \int \frac{\sqrt{x} dx}{\sqrt[4]{x^3+1}} \dots\dots\dots (\text{resp. } \frac{4}{3} [x^{3/4} - \log(1+x^{3/4})] + C)$$

$$2. \int \frac{(\sqrt{x^3} - \sqrt[3]{x}) dx}{6\sqrt[4]{x}} \dots\dots\dots (\text{resp. } \frac{2}{27}x^{9/4} - \frac{2}{13}x^{13/12}) + C)$$

$$3. \int \frac{(\sqrt[6]{x} + 1) dx}{\sqrt[6]{x^7} + \sqrt[4]{x^5}} \dots\dots\dots (\text{resp. } -\frac{6}{\sqrt[6]{x}} + \frac{12}{\sqrt[12]{x}} + 2 \log x - 24 \log(\sqrt[12]{x} + 1) + C)$$
